Accurate Calculation of Bit Error Rates in Optical Fiber Communications Systems

presented by

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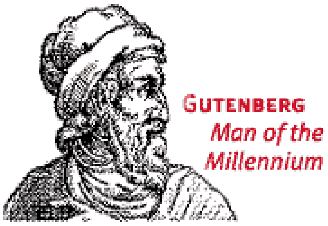
Invention of the Printing Press

~ 1452 – 1455



Accuracy

- Of mathematical models: Physics → Equations
- Of solution algorithms: Equations → Solutions

Focus here is on algorithms

Basic Difficulty

Nonlinearity in transmission; nonlinearity in receiver
⇒ Traditional analytical approaches do not work

Lower error rates (~10^{-15} in many cases)
⇒ Standard Monte Carlo methods do not work

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Validation

- Deterministic methods;
  Faster ↔ Approximate

- Statistical (biasing Monte Carlo) methods;
  Slower ↔ Arbitrarily accurate

Additional difficulty

- System complexity:
  transmitter + receiver + error-correction
  must be analyzed together

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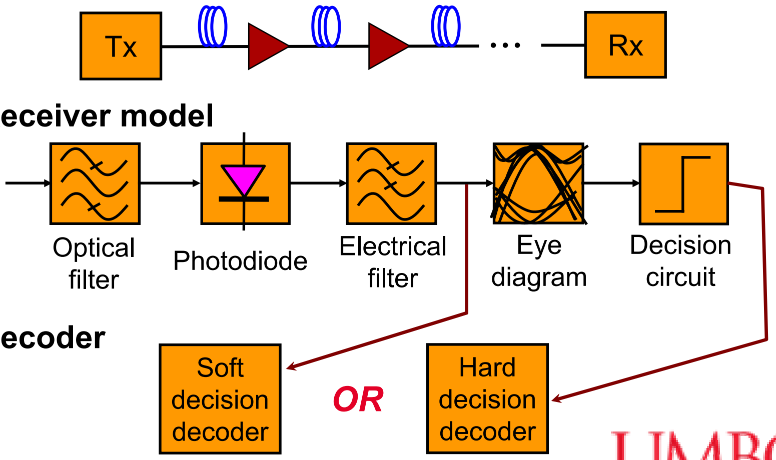
Basic Transmission System

Transmission line



Receiver model

Decoder

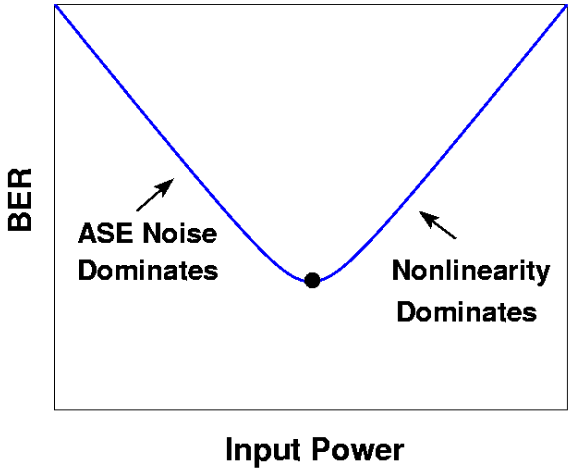
Receiver

Input Multivariate Gaussian Noise + Signal (any OOK format)

⇒ χ^2 distribution of voltage

Lee and Shim, JLT 1994
Bosco *et al*., IEEE PTL 2000
Forestieri *et al*., JLT 2000
Holzlöhner *et al*., JLT 2002
Carlsson *et al*., OFC 2003

BER vs. Input Power



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Effects of Nonlinearity in Transmission

- Noise-signal interactions
- Pattern dependences
  - Complex in WDM systems

Focus first on noise-signal interactions!

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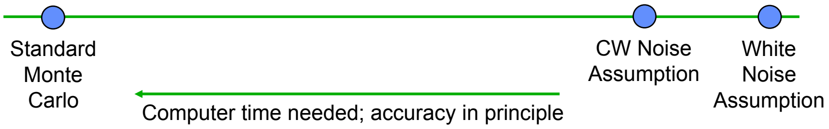
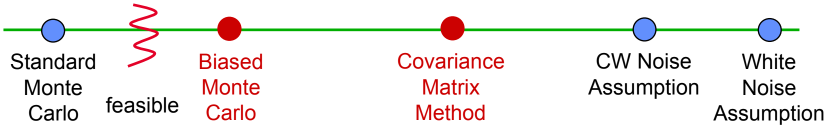
Traditional Methods



- Standard Monte Carlo: ~ 10^{12} NF
  - — randomness yields intrinsic errors
- White noise assumption: ~ 1 NF
  - — just plain wrong in many long-haul systems
- CW noise assumption: ~ 10 NF
  - — takes into account parametric pumping

NF = noise-free simulation

Our Approaches



- Covariance matrix method: ~ 10^2 NF
  - — assumes noise-noise beating is negligible in transmission

    (with caveats!)
- Biased Monte Carlo: ~ 10^5 NF
  - — keeps everything in principle!

NF = noise-free simulation

Covariance Matrix Method

Basic assumption:

Noise-noise beating in transmission is negligible once phase noise is separated

Consequences:

Optical noise distribution is multivariate Gaussian

The distribution is completely determined by the noise covariance matrix

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Covariance Matrix Method

Other points:

The covariance matrix can be calculated deterministically

Multivariate Gaussian distributed optical noise maps to a generalized χ^2 distributed current

The whole distribution function can be calculated deterministically!

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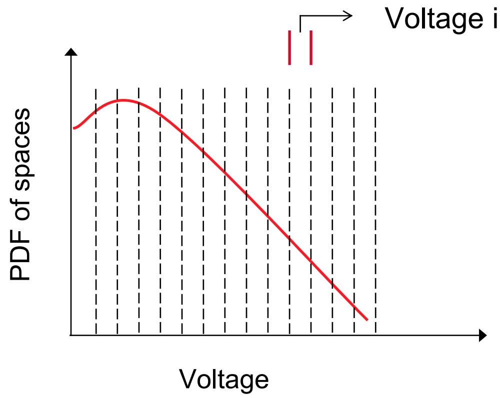
Multicanonical Monte Carlo (MMC)

Goal: To obtain an equal number of realizations in each voltage interval in the region of interest



Multicanonical Monte Carlo (MMC)

Procedure (a bit simplified) :

Do standard Monte Carlo based on Metropolis algorithm

In step i:

- $z_{\text{prov}}^{i+1} = z^i + \Delta z^i$ (Δz^i is randomly chosen)
  [z is a point in the configuration space]
- Calculate $\rho^{i+1} \equiv \rho(z_{\text{prov}}^{i+1})$
  [ρ is the probability density]
- Accept provisional step with probability $\min(1, \rho^{i+1} / \rho^i)$
- If step accepted : $z^{i+1} = z_{\text{prov}}^{i+1}$
  If step rejected : $z^{i+1} = z^i$
- Increment k th voltage bin by 1

Multicanonical Monte Carlo (MMC)

Estimate $P_k^1 = N_k^1 / N_{\text{total}}^1$
[P_k^1 is the probability that the voltage is in bin k]

Repeat the Metropolis algorithm with the change:

- Accept provisional step with probability
  $\min(1\,,\, P_{k,i}^1 \rho^{i+1} / P_{k,i+1}^1 \rho^i)$

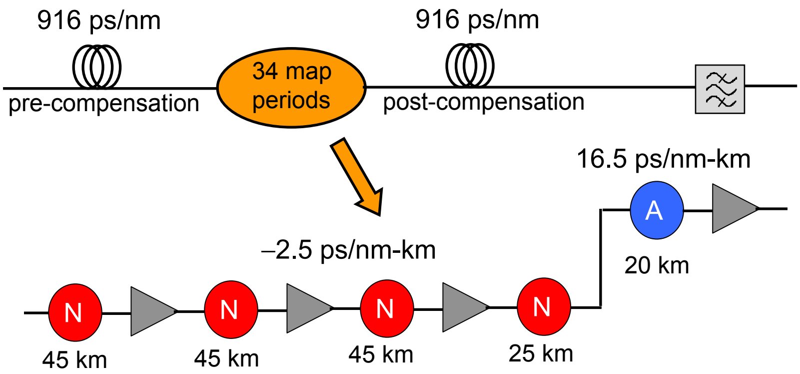
Estimate $P_k^2 = C^1 P_k^1 N_k^2 / N_{\text{total}}^2$ [C^1 = normalization constant]

Iterate until convergence

No a priori knowledge of how to bias is needed!

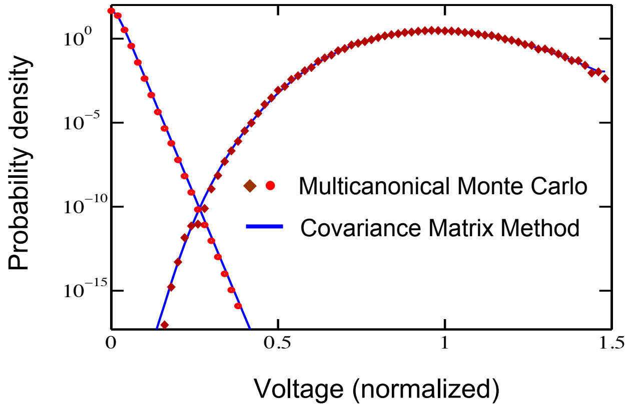
Chirped RZ System

Submarine single-channel 10 Gb/s CRZ system, 6120 km



Nonlinear scale length: 1960 km
System length: ~ 3 nonlinear scale lengths

Results



Covariance matrix method and multicanonical Monte Carlo agree perfectly over 15 orders magnitude!*

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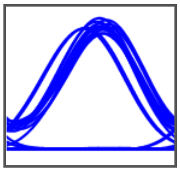
*R. Holzlöhner and C. R. Menyuk, Opt. Lett. 28, 1894 (2003)

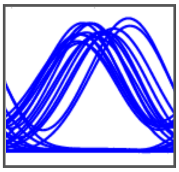
Interchannel pattern dependences

Simulation results with the same bit pattern in the center channel but different bit patterns in the other channels:

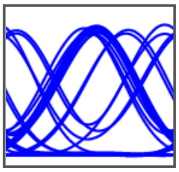
pattern 1



pattern 2



pattern 3



$\Delta\Omega$=50 GHz

L=5000 km

Nonlinear penalty is bit-pattern dependent

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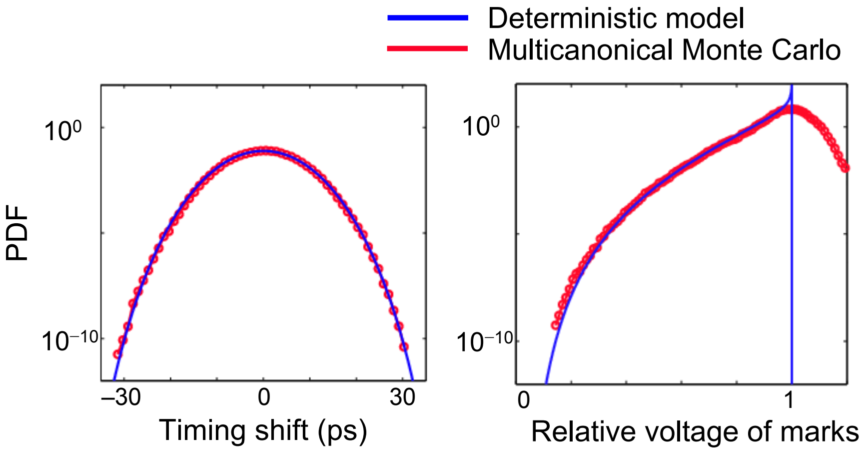
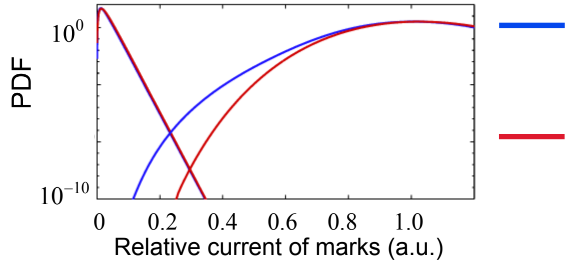
Voltage PDF due to nonlinearity



BER with pattern dependencies

Compute* $p_{\text{noise}}(I,t)$ and convolve with timing shift PDF:

$$p_{\text{total}}(I,t) = p_{\text{noise}}(I,t) * p_{\Delta T}(t)$$



$p_{\text{total}}(I,t)$
BER=5.8×10^{-12}

$p_{\text{noise}}(I,t)$
BER=3.3×10^{-15}

*Forestieri, *J. Lightwave Technol*. No.11, 2000
Holzlöhner *et al.* , *PTL*. No.8, 2002

Error Correcting Codes

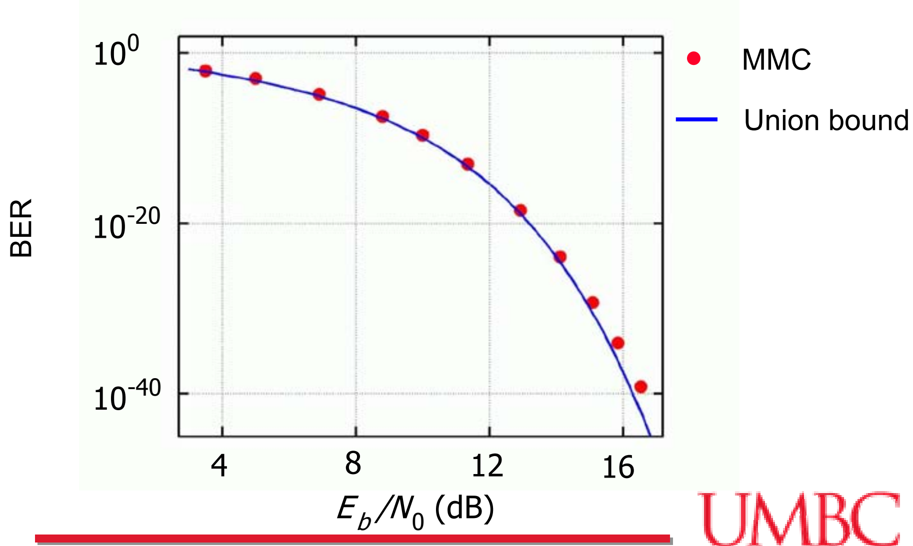
Low density parity check code

Union bound gives an upper bound for the BER of the maximum-likelihood decoder

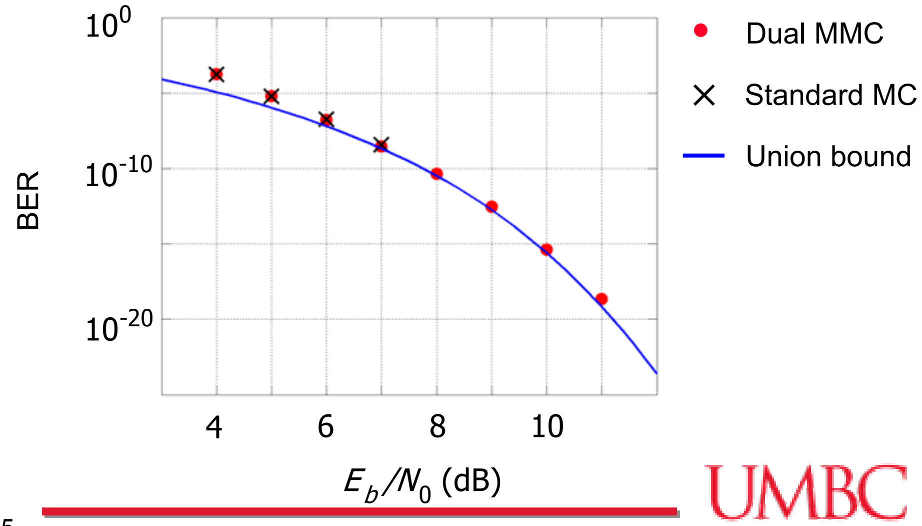
Multicanonical Monte Carlo can be used with a modified procedure:

- Calculate probability of errors vs. voltage (standard)
  [Produces high variance at low voltages with errors]
- Calculate probability of errors vs. voltage (only steps that produce errors are accepted)
  [Produces low variance at low voltages]

BER vs. SNR



BER vs. SNR



Conclusions

Important issues remain

- Combining noise, pattern dependences, error correction
- Validating simple fast approaches
- Formats besides RZ
- Experimental validation

Methods that allow accurate calculations of BER — based on first principles — have been developed

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CW noise method

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Covariance Matrix Method

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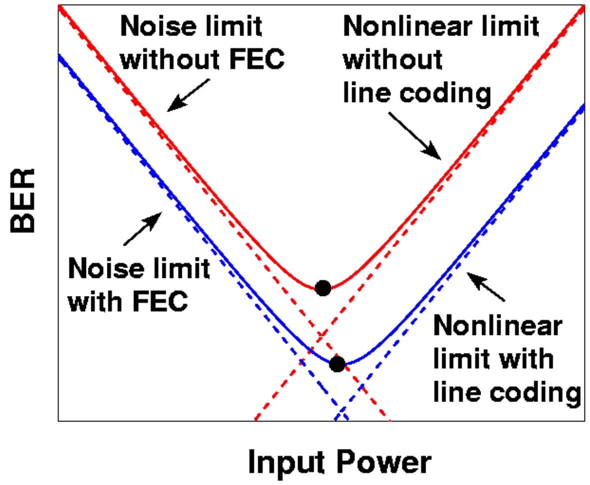
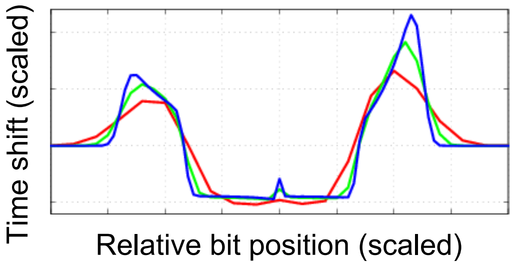
BER vs. Input Power



Data-pattern dependences

CRZ systems:

Inter-channel XPM-induced timing jitter dominates



Scaling:

Amplitude ~$1/\Delta\Omega^2$

Width ~$\Delta\Omega$

Add time shifts

Use receiver model to find penalties

Voltage PDF due to nonlinearity