Accurate Calculation of Bit Error Rates in Optical Fiber Communications Systems

presented by
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Invention of the Printing Press

~ 1452 – 1455

Accuracy

• Of mathematical models: Physics → Equations
• Of solution algorithms: Equations → Solutions

Focus here is on algorithms
Basic Difficulty

Nonlinearity in transmission; nonlinearity in receiver
⇒ Traditional analytical approaches do not work

Lower error rates (~10^{-15} in many cases)
⇒ Standard Monte Carlo methods do not work

Validation

- Deterministic methods;
  Faster ⇐ Approximate

- Statistical (biasing Monte Carlo) methods;
  Slower ⇐ Arbitrarily accurate

Additional difficulty
- System complexity:
  transmitter + receiver + error-correction
  must be analyzed together
**Basic Transmission System**

Transmission line

Tx → … → Rx

Receiver model

Optical filter → Photodiode → Electrical filter → Eye diagram → Decision circuit

Decoder

<table>
<thead>
<tr>
<th>Soft decision decoder</th>
<th>Hard decision decoder</th>
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**Receiver**

Input Multivariate Gaussian Noise + Signal (any OOK format)

⇒ $\chi^2$ distribution of voltage

Lee and Shim, JLT 1994  
Bosco et al., IEEE PTL 2000  
Forestieri et al., JLT 2000  
Holzlöhner et al., JLT 2002  
Carlsson et al., OFC 2003
Effects of Nonlinearity in Transmission

- Noise-signal interactions

- Pattern dependences
  - Complex in WDM systems

*Focus first on noise-signal interactions!*
Traditional Methods

- **Standard Monte Carlo:** ~ $10^{12}$ NF
  - randomness yields intrinsic errors
- **White noise assumption:** ~ 1 NF
  - just plain wrong in many long-haul systems
- **CW noise assumption:** ~ 10 NF
  - takes into account parametric pumping

NF = noise-free simulation

Our Approaches

- **Covariance matrix method:** ~ $10^2$ NF
  - assumes noise-noise beating is negligible in transmission
    *(with caveats!)*
- **Biased Monte Carlo:** ~ $10^5$ NF
  - keeps everything in principle!

NF = noise-free simulation
Covariance Matrix Method

Basic assumption:

*Noise-noise beating in transmission is negligible once phase noise is separated*

Consequences:

Optical noise distribution is multivariate Gaussian

The distribution is completely determined by the noise covariance matrix

Covariance Matrix Method

Other points:

The covariance matrix can be calculated deterministically

Multivariate Gaussian distributed optical noise maps to a generalized $\chi^2$ distributed current

*The whole distribution function can be calculated deterministically!*
**Multicanonical Monte Carlo (MMC)**

**Goal:** To obtain an equal number of realizations in each voltage interval in the region of interest

**Procedure (a bit simplified):**

Do standard Monte Carlo based on Metropolis algorithm

In step $i$:

- $z_{\text{prov}}^{i+1} = z^i + \Delta z^i$ (\(\Delta z^i\) is randomly chosen)
  
  [\(z\) is a point in the configuration space]

- Calculate $\rho^{i+1} \equiv \rho(z_{\text{prov}}^{i+1})$
  
  [\(\rho\) is the probability density]

- Accept provisional step with probability $\min(1, \frac{\rho^{i+1}}{\rho^i})$

- If step accepted: $z^{i+1} = z_{\text{prov}}^{i+1}$
  
  If step rejected: $z^{i+1} = z^i$

- Increment $k$ th voltage bin by 1
Multicanonical Monte Carlo (MMC)

Estimate $P^1_k = N^1_k / N^1_{\text{total}}$

[$P^1_k$ is the probability that the voltage is in bin $k$]

Repeat the Metropolis algorithm with the change:

- Accept provisional step with probability
  \[ \min(1, P^1_{k,i} P^{i+1}_{k,i-1} / P^1_{k,i-1} P^i_{k,i}) \]

Estimate $P^2_k = C^1 P^1_k N^2_k / N^2_{\text{total}}$  \[ C^1 = \text{normalization constant} \]

Iterate until convergence

*No a priori knowledge of how to bias is needed!*

Chirped RZ System

Submarine single-channel 10 Gb/s CRZ system, 6120 km

916 ps/nm

pre-compensation 34 map periods post-compensation

916 ps/nm

16.5 ps/nm-km

-2.5 ps/nm-km

20 km

45 km

25 km

Nonlinear scale length: 1960 km

System length: \~ 3 nonlinear scale lengths
Results

Covariance matrix method and multicanonical Monte Carlo agree perfectly over 15 orders magnitude!*

Interchannel pattern dependences

Simulation results with the same bit pattern in the center channel but different bit patterns in the other channels:

\[ \Delta \Omega = 50 \text{ GHz} \]
\[ L = 5000 \text{ km} \]

Nonlinear penalty is bit-pattern dependent
**Voltage PDF due to nonlinearity**

Deterministic model

Multicanonical Monte Carlo

**PDF**

Timing shift (ps)

Relative voltage of marks

**BER with pattern dependencies**

Compute $p_{\text{noise}}(I,t)$ and convolve with timing shift PDF:

$$p_{\text{total}}(I,t) = p_{\text{noise}}(I,t) * p_{\Delta T}(t)$$

PDF

Relative current of marks (a.u.)

$*$Forestieri, J. Lightwave Technol. No.11, 2000
Holzlöhner et al., PTL. No.8, 2002
**Error Correcting Codes**

Low density parity check code

Union bound gives an upper bound for the BER of the maximum-likelihood decoder

Multicanonical Monte Carlo can be used with a modified procedure:

- Calculate probability of errors vs. voltage **(standard)**
  [Produces high variance at low voltages with errors]

- Calculate probability of errors vs. voltage **(only steps that produce errors are accepted)**
  [Produces low variance at low voltages]

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**BER vs. SNR**

- **MMC**
- **Union bound**

![BER vs. SNR graph]

- BER vs. $E_b/N_0$ (dB)
Conclusions

Important issues remain
   — Combining noise, pattern dependences, error correction
   — Validating simple fast approaches
   — Formats besides RZ
   — Experimental validation

Methods that allow accurate calculations of BER — based on first principles — have been developed
References

CW noise method


References

Covariance Matrix Method


References

Receiver models


References

Collision-induced timing jitter in RZ systems


Multicanonical Monte Carlo Method


LDPC Codes


**BER vs. Input Power**

- Noise limit without FEC
- Nonlinear limit without line coding
- Noise limit with FEC
- Nonlinear limit with line coding

**Data-pattern dependences**

**CRZ systems:**
Inter-channel XPM-induced timing jitter dominates

**Scaling:**
- Amplitude $\sim 1/\Delta \Omega^2$
- Width $\sim \Delta \Omega$

Add time shifts
Use receiver model to find penalties
Voltage PDF due to nonlinearity

- Multicanonical Monte Carlo
- Reduced Model

PDF vs. Relative voltage of marks graph.