Recent Advances in Fiber-Optic Parametric Amplifiers

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Introduction

- Industrial revolution of 19th century was followed by Information revolution during the 1990s.
- Fiber-optic revolution is a natural consequence of the Internet growth.
Five Generations

- 0.8-μm systems (1980); Graded-index fibers
- 1.3-μm systems (1985); Single-mode fibers
- 1.55-μm systems (1990); Single-mode lasers
- WDM systems (1996); Optical amplifiers
- L and S bands (2001); Raman amplification
Parametric Amplifiers

Fiber-optic Parametric Amplifiers (FOPAs) exhibit

- Large signal amplification (>40 dB)
- Wide gain bandwidth (>50 nm)
- Nearly uniform gain spectrum (<1 dB variations)
- Relatively low noise (noise figure <4 dB)

FOPAs can be used for

- Optical amplification
- Wavelength conversion
- Phase conjugation
- Ultrafast signal processing
Four-Wave Mixing (FWM)

- FWM is a nonlinear process that transfers energy of pumps to signal and idler waves.

- FWM requires conservation of (notation: $E = \text{Re}[A \exp(i\beta z - i\omega t)]$)
  - Energy \[\omega_1 + \omega_2 = \omega_3 + \omega_4\]
  - Momentum \[\beta_1 + \beta_2 = \beta_3 + \beta_4\]

- Degenerate FWM: Single pump ($\omega_1 = \omega_2$).
Single- and Dual-Pump FOPAs

- Pump close to fiber’s ZDWL
- Wide but nonuniform gain spectrum with a dip

- Pumps at opposite ends
- Much more uniform gain
- Lower pump powers (∼0.5 W)
Dual-Pump FOPAs

- Typical output spectrum of dual-pump FOPAs.
- Multiple idlers generated when signal is launched with two pumps.
- Two pumps act together to generate an idler (nondegenerate FWM).
- Each pump also produces its own idler through degenerate FWM.
Theory of FOPAs

- Full problem quite complicated (4 coupled nonlinear equations)

- Undepleted-pump approximation $\implies$ two linear coupled equations:

$$\frac{dA_3}{dz} = \frac{i}{2} \kappa A_3 + 2i\gamma \sqrt{P_1 P_2} A_4^*$$

$$\frac{dA_4}{dz} = \frac{i}{2} \kappa A_4 + 2i\gamma \sqrt{P_1 P_2} A_3^*$$

- Phase mismatch: $\kappa = \beta_3 + \beta_4 - \beta_1 - \beta_2 + \gamma (P_1 + P_2)$.

- Nonlinear parameter: $\gamma = n_2 \omega_0 / (ca_{eff}) \sim 2-20 \text{ W}^{-1}/\text{km}$.

- Signal power $P_3$ and Idler power $P_4$ are much smaller than pump powers $P_1$ and $P_2$ ($P_n = |A_n|^2$).
Simple Theory of FOPAs

- Signal and idler powers obey the same equation \( A_3 = \sqrt{P_3}e^{i\phi_3} \)
  \[
  \frac{dP_3}{dz} = \frac{dP_4}{dz} = 4\gamma\sqrt{P_1P_2P_3P_4}\sin\theta
  \]

- Phase equation \( (\theta = \phi_3 + \phi_4 - \phi_1 - \phi_2) \)
  \[
  \frac{d\theta}{dz} = \kappa + 2\gamma(P_3 + P_4)\sqrt{\frac{P_1P_2}{P_3P_4}}\cos\theta
  \]

- In the case of perfect phase matching \( (\kappa = 0) \), idler is generated such that \( \theta = \pi/2 \), and \( \theta \) remains fixed.

- When no idler power is launched initially, \( \theta = \pi/2 \) even if phase matching is not perfect \( (\kappa \neq 0) \).
Gain Spectrum

- Signal amplification factor or FOPA gain $G$:

$$ G(\omega) = \frac{P_3(L, \omega)}{P_3(0, \omega)}. $$

- Coupled signal and idler equations can be written in a matrix form:

$$ \frac{\partial}{\partial z} \begin{pmatrix} A_3 \\ A_4^* \end{pmatrix} = \frac{i}{2} \begin{pmatrix} \kappa(\omega) & 4\gamma\sqrt{P_1P_2} \\ -4\gamma\sqrt{P_1P_2} & -\kappa(\omega) \end{pmatrix} \begin{pmatrix} A_3 \\ A_4^* \end{pmatrix}. $$

- This matrix equation is easily solved for a FOPA of length $L$ to obtain

$$ G(\omega) = \left[ 1 + \left( 1 + \frac{\kappa^2(\omega)}{4g^2(\omega)} \right) \sinh^2[g(\omega)L] \right]. $$

- Parametric gain: $g(\omega) = \sqrt{4\gamma^2P_1P_2 - \kappa^2(\omega)/4}$. 
Phase-Matching Condition

- Total phase mismatch \( \kappa = \Delta \beta_L + \Delta \beta_{NL} \)
- Linear phase mismatch: \( \Delta \beta_L(\omega) < 0 \) is required.
- Nonlinear phase mismatch: \( \Delta \beta_{NL}(P_1, P_2) = \gamma P_1 P_2 > 0 \)

FOPA length = 0.5 km
\( P_1 = P_2 = 0.5 \, W \)
\( \gamma = 10 \, W^{-1}/km \)
\( \beta_3 = 0.1 \, ps^3/km \)
\( \beta_4 = 10^{-4} \, ps^4/km \)
Single pump (blue curves)
Two pumps (red curves)
Applications of FOPAs

- Parametric amplification
- Optical Phase Conjugation
- Wavelength Conversion
- Optical Sampling
- Time-Domain Demultiplexing
- Packet and Bit-Level Switching
- All-Optical Regeneration
Parametric amplification

- **SBS problem** $\implies$ Modulate pump phases at bit rates $\sim 5$ Gb/s.
- **Amplifier noise** $\implies$ Use narrowband optical filters.
- **Highly nonlinear fiber**: Narrow core enhances $\gamma$ and $G$. 
Comparison with Theory

- FWM theory predicts bandwidths $> 50$ nm under ideal conditions.
- Maximum bandwidth realized experimentally is around 40 nm.
- Fit to data requires inclusion of the Raman effects.
Optical Phase Conjugation

- FWM generates an idler wave during parametric amplification.
- Its phase is complex conjugate of the signal field ($A_4 \propto A_3^*$).
- Phase conjugation can be used for dispersion compensation by placing a parametric amplifier midway.
- Basic idea patented in 1979; first demonstration in 1993.
- Phase conjugation can also reduce timing jitter in lightwave systems.
Experimental Results

- Pump wavelength coincided with the zero-dispersion wavelength.
- 10-Gb/s signal could be transmitted over 360 km of standard fiber.
- Distance limited to 32 km without phase conjugation.
Wavelength Conversion

- FOPAs can transfer data to a different wavelength.
- A CW pump beam is launched into the fiber together with the signal channel.
- Pump wavelength is chosen halfway from the desired shift.
- FWM transfers the data from signal to the idler wave at the new wavelength.
Multichannel Wavelength Conversion

- Islam et al., IEEE JSTQE 8, 527 (2002).
- 860-mW peak power pump at 1532 nm; 315-m-long fiber.
- 32 channels converted into S band with 4.7 dB conversion efficiency.
Optical Sampling

- Short samplings pulses (~1 ps) act as the pump.
- Idler pulse width comparable to pump pulses.
- Eye diagrams produced up to 300 Gb/s.
- Temporal resolution of 1.6 ps possible; Li et al., PTL 13, 987 (2001).
Time-Domain Demultiplexing

- FOPAs can be used to demultiplex OTDM channels.
- Pump is an optical clock at a different wavelength.
- Only signal pulses overlapping with clock pulses can generate an idler pulse.
- An optical filter blocks the pump and other channels.
- This method can work at bit rates of 500 Gb/s or more; Morioka et al., Electron. Lett. 32, 832 (1996).
Packet Switching of 40-Gb/s Signal

- Experiment by Lin, Radic, and Agrawal (2005).
- L-band pump modulated to produce pulses of packet size.
- FWM occurs only over the packet duration
- Both signal and idler contain bits belonging to one packet.
- This technique can be adopted to any packet size.
Bit-Level Switching at 40 Gb/s

- Experiment by Lin, Radic, and Agrawal (2005).

- L-band pump contains 1-bit-wide pulses.

- FWM occurs only over the bit duration.

- A single signal bit can be switched with this technique.

- Idler wave contains only switched bits in time slots selected by the pump.
Optical Regeneration

- All-optical regeneration demonstrated with dual pump FOPAs.
- Up to 20 dB improvement in signal-to-noise ratio (SNR).
Performance Limiting Factors

Several factors affect FOPA performance:

- SNR degradation due to pump-phase modulation; PM-to-AM conversion through fiber dispersion.
- Noise added to pumps during their amplification degrades SNR of both signal and idler.
- Dispersion fluctuations in optical fibers.
- Polarization sensitivity of the FWM process.
- Birefringence of single-mode fibers (PMD).
- Raman-induced noise and power transfer.
PM-to-AM Conversion of Pumps

- Pump phases modulated to suppress stimulated Brillouin scattering.
- Phase modulation converted to amplitude modulation by dispersion.
- Signal gain sensitive to changes in pump power:
  \[ G \approx \frac{1}{4} \exp(gL) \quad g = 2\gamma\sqrt{P_1P_2}. \]
- Signal and idler SNR degraded considerably because of pump power variations.
- Walk-off effects reduce the impact of pump-phase modulation.
- Effective parametric gain: \( \delta = \text{walk-off parameter} \)
  \[ \bar{g} = \frac{1}{L} \int_0^L 2\gamma[P_1(z, \tau - z\delta) P_2(z, \tau - z\delta)]^{1/2}dz. \]
PM-to-AM Conversion

- Signal power variation larger for shorter rise times.
- Group-velocity mismatch reduces the effect considerably.

Length = 1 km
$\gamma = 4.2 \text{ W}^{-1}/\text{km}$
$P_1 = P_2 = 0.5 \text{ W}$
$\beta_3 = 0.1 \text{ ps}^3/\text{km}$
$\beta_4 = 8 \times 10^{-5} \text{ ps}^4/\text{km}$
$T_r = 33 \text{ ps (blue)}$
$T_r = 14 \text{ ps (red)}$
Signal SNR

- SNR degrades with higher modulation rates and shorter rise times
- Walk-off effects improve the SNR at high bit rates.
• Erbium-doped fiber amplifiers add noise to pump.
• Noise is quantified through relative intensity noise (RIN)
• How does pump RIN affect the amplified signal and idler?
Dual-Pump FOPA Equations

\[
\frac{\partial A_1}{\partial z} = i\beta_0(\omega_1)A_1 - \frac{1}{v_{g1}} \frac{\partial A_1}{\partial t} + i\gamma(|A_1|^2 + 2|A_2|^2)A_1 \\
\frac{\partial A_2}{\partial z} = i\beta_0(\omega_2)A_2 - \frac{1}{v_{g2}} \frac{\partial A_2}{\partial t} + i\gamma(|A_2|^2 + 2|A_1|^2)A_2 \\
\frac{\partial A_3}{\partial z} = i\beta_0(\omega_3)A_3 - \frac{1}{v_{g3}} \frac{\partial A_3}{\partial t} + 2i\gamma(|A_1|^2 + |A_2|^2)A_3 + 2i\gamma A_1A_2A_4^* \\
\frac{\partial A_4}{\partial z} = i\beta_0(\omega_4)A_4 - \frac{1}{v_{g4}} \frac{\partial A_4}{\partial t} + 2i\gamma(|A_1|^2 + |A_2|^2)A_4 + 2i\gamma A_1A_2A_3^*
\]

This set of equations is solved numerically to calculate:

- \( RIN_p(\omega, 0) = \frac{1}{\langle P_1 \rangle^2} \int_{-\infty}^{\infty} \langle \delta P_1(t) \delta P_1(t + \tau) \rangle \exp(-i\omega \tau) d\tau \)
- \( RIN_s(\omega, L) = \frac{1}{\langle P_3 \rangle^2} \int_{-\infty}^{\infty} \langle \delta P_3(t) \delta P_3(t + \tau) \rangle \exp(-i\omega \tau) d\tau \)
- RIN Enhancement factor: \( F_r(\omega) = RIN_s(\omega, L) / RIN_p(\omega, 0) \).
RIN Spectra

- At low frequencies, signal RIN is enhanced by 15 dB.
- At high frequencies, RIN is reduced because of walk-off effects.
Optical SNR

- Signal SNR is degraded considerably because of pump RIN.
- A narrowband optical filter helps but problem persists.
Dispersion Fluctuations

- Fiber-core diameter is not uniform along fiber length.
- Zero-dispersion wavelength (ZDWL) fluctuates along $z$.
- Even 1% variations in core diameter cause 4–6 nm change in ZDWL.
- Length scale of ZDWL fluctuations is typically $\sim 10$ m.
- Phase mismatch $\kappa$ depends on the ZDWL; it also varies randomly along the FOPA length.
- Output signal power can vary considerably because of ZDWL fluctuations for otherwise identical FOPAs.
Signal Gain Fluctuations

\[ L = 1 \text{ km} \]
\[ \gamma = 4.2 \, \text{W}^{-1}\text{km}^{-1} \]
\[ P_1 = P_2 = 0.5 \, \text{W} \]
\[ \beta_3 = 0.1 \, \text{ps}^3/\text{km} \]
\[ \beta_4 = 8 \times 10^{-5} \, \text{ps}^4/\text{km} \]

- Pumps separated by 100 nm in numerical simulations.
- Each curve corresponds to a fiber with a different ZDWL distribution but the same average value.
- Central portion of the gain spectrum is affected most.
Compromise: Trade Bandwidth for Flatness

- Pumps separated by 50 nm; other parameters unchanged.
- Gain is nearly uniform in spite of dispersion fluctuations.
- FOPA bandwidth is reduced to below 45 nm.
Polarization Effects

- FWM process in optical fibers is polarization sensitive.


- It makes use of Jones matrices and rotation of Stokes vectors on the Poincaré sphere.

- Parametric gain depends on the relative polarization of two pumps.

- FOPA gain becomes independent of signal polarization for orthogonally polarized pumps (in the absence of PMD).

- Linearly but orthogonally polarized pumps used in practice.
Linear versus Circular Polarization

- Parametric gain varies with the ellipticity of pumps.
- Linearly polarized pumps provide only 50% gain even for perfect phase matching.
- Circularly polarized pumps provide the best performance.
Polarization-Mode Dispersion

- A single-mode fiber supports two orthogonally polarized modes.
- Two modes degenerate for a perfect fiber \((n_x = n_y)\).
- In practice, \(|n_x - n_y| \sim 10^{-7}\) on average, but it changes along the fiber in a random fashion.
- State of polarization of light evolves randomly at a rate that depends on its frequency (PMD).

![Evolution of polarizations of the four interacting fields along fiber.](image-url)
PMD-Induced Gain Fluctuations

• Gain fluctuates by a large amount from fiber to fiber.
• For a given fiber, gain can also fluctuate with time.
• Gain uniformity is degraded in both cases.
Average Gain Spectra

$D_p$ characterizes strength of PMD effects.

Typically, $D_p = 0.1 \text{ ps/km}^{1/2}$ for modern fibers.

- Average gain is reduced because of birefringence fluctuations.
- Gain uniformity is degraded for large PMD.
- Uniformity can be improved by reducing pump separation (resulting in a lower gain bandwidth).
Polarization-Dependent Gain (PDG)

- Gain is sensitive to input SOP of signal polarization even if orthogonally polarized pumps are used.
Conclusions

• FOPA is a new kind of fiber-based parametric device.

• It can act as an optical amplifier and provide $>$30 dB gain over a bandwidth of $>$40 nm.

• It can also be used for wavelength conversion, phase conjugation, and many other signal-processing applications.

• Dual-pump FOPAs require two pumps with $\sim$0.5 W power.

• Power and phase fluctuations of pumps affect the signal SNR and should be minimized.

• PMD and dispersion fluctuations also affect FOPA performance.

• FOPAs constitute an active topic of current research.